

# ON THE INFLUENCE OF GRAVITATIONAL RADIATION ON A GYROSCOPE

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## Abstract

We calculate the precession of a gyroscope at rest in a Bondi space-time. It is shown that, far from the source, the leading term in the rate of precession of the gyroscope is simply expressed through the news function of the system, and vanishes if and only if there is no news. Rough estimates are presented, illustrating the order of magnitude of the expected effect for different scenarios. It is also shown from the next order term ( $\frac{1}{r^2}$ ) that non-radiative (but time dependent) spacetimes will produce a gyroscope precession of that order, providing thereby “observational” evidence for the violation of the Huygens’s principle.

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# 1 Introduction

The theoretical description and the experimental observation of gravitational radiation are among the most relevant challenges confronting general relativity.

A great deal of work has been done so far in order to provide a consistent framework for the study of such phenomenon. Also, since Weber's pioneering work [1] important collaboration efforts have been carried on, and are now under consideration, to put in evidence gravitational waves (see [2], [3], [4],[5], and references therein).

It is the purpose of this work to evaluate the influence of gravitational radiation on a gyroscope. This idea is not new, in fact some years ago, Chaboyer and Henriksen [6] put forward the possibility of detecting gravitational radiation by means of an orbital laser gyroscope. In such experiment the presence of gravitational radiation is brought out by the differential effect that radiation has on the paths of the photons in the rotating frame of reference.

In this work we shall calculate the rate of precession of a gyroscope in the field of gravitational radiation. To do so we shall use the Bondi's formalism [7] which has, among other things, the virtue of providing a clear and precise criterion for the existence of gravitational radiation (see also [8]). Namely, if the news function is zero over a time interval, then there is no radiation during that interval. Also, the present approach has the advantage of providing a very simple expression linking an "observable" (at least in principle) quantity (the rate of precession of a gyroscope) with the emission rate of gravitational radiation.

The formalism has as its main drawback [9] the fact that it is based on a series expansion which could not give closed solutions and which raises unanswered questions about convergence and appropriateness of the expansion.

However since we shall assume the gyroscope to be very far from the source, we shall use in our calculations only the leading terms in the expansion of metric functions. Furthermore, since the source is assumed to radiate during a finite interval, then no problem of convergence appears [10].

We shall see that the leading term of the rate of precession of the gyroscope ( $\Omega$ ) is expressed through the news function in such a way that it will vanish if and only if there is no news (no radiation).

In the special case of the quadrupole radiation (in the linear approximation), the rate of precession may be expressed through the third time

derivative of the quadrupole moment, or alternatively, through the rate of loss of the mass function.

Next we present the order  $(\frac{1}{r^2})$  of  $\Omega$ . As we shall see, it contains terms with news, together with a time dependent term not involving news. This last term represents the class of non-radiative motions discussed by Bondi [7] and may be thought to correspond to the tail of the wave, appearing after the radiation process [9]. The obtained expression allows for “measuring” (in a gedanken experiment, at least) the wave-tail field. This in turn implies that observing the gyroscope, for a period of time from an initial static situation until after the vanishing of the news, should allow for an unambiguous identification of a gravitational radiation process.

In the next section we briefly present the Bondi’s formalism. The expression for the rate of precession of the gyroscope in the Bondi metric is calculated in sections 3 and 4, and estimates for the rate of precession in different scenarios are presented in section 5. Finally the results are discussed in the last section.

## 2 The Bondi’s Formalism

The general form of an axially symmetric asymptotically flat metric given by Bondi is [7]

$$ds^2 = \left( \frac{V}{r} e^{2\beta} - U^2 r^2 e^{2\gamma} \right) du^2 + 2e^{2\beta} du dr + 2Ur^2 e^{2\gamma} du d\theta - r^2 \left( e^{2\gamma} d\theta^2 + e^{-2\gamma} \sin^2 \theta d\phi^2 \right) \quad (1)$$

where  $V, \beta, U$  and  $\gamma$  are functions of  $u, r$  and  $\theta$ .

We number the coordinates  $x^{0,1,2,3} = u, r, \theta, \phi$  respectively.  $u$  is a timelike coordinate such that  $u = \text{constant}$  defines a null surface. In flat spacetime this surface coincides with the null light cone open to the future.  $r$  is a null coordinate ( $g_{rr} = 0$ ) and  $\theta$  and  $\phi$  are two angle coordinates (see [7] for details).

Regularity conditions in the neighborhood of the polar axis ( $\sin \theta = 0$ ), implies that as  $\sin \theta \rightarrow 0$

$$V, \beta, U / \sin \theta, \gamma / \sin^2 \theta \quad (2)$$

each equals a function of  $\cos \theta$  regular on the polar axis.

The four metric functions are assumed to be expanded in series of  $1/r$ , then using field equations Bondi gets

$$\gamma = c(u, \theta)r^{-1} + \left(C(u, \theta) - \frac{1}{6}c^3\right)r^{-3} + \dots \quad (3)$$

$$U = -(c_\theta + 2c \cot \theta)r^{-2} + \dots \quad (4)$$

$$\begin{aligned} V &= r - 2M(u, \theta) \\ &- \left(N_\theta + N \cot \theta - c_\theta^2 - 4cc_\theta \cot \theta - \frac{1}{2}c^2(1 + 8 \cot^2 \theta)\right)r^{-1} + \dots \end{aligned} \quad (5)$$

$$\beta = -\frac{1}{4}c^2r^{-2} + \dots \quad (6)$$

where letters as subscripts denote derivatives, and

$$4C_u = 2c^2c_u + 2cM + N \cot \theta - N_\theta \quad (7)$$

The three functions  $c$ ,  $M$  and  $N$  are further related by the supplementary conditions

$$M_u = -c_u^2 + \frac{1}{2}(c_{\theta\theta} + 3c_\theta \cot \theta - 2c)_u \quad (8)$$

$$-3N_u = M_\theta + 3cc_{u\theta} + 4cc_u \cot \theta + c_u c_\theta \quad (9)$$

In the static case  $M$  equals the mass of the system whereas  $N$  and  $C$  are closely related to the dipole and quadrupole moment respectively.

Next, Bondi defines the mass  $m(u)$  of the system as

$$m(u) = \frac{1}{2} \int_0^\pi M(u, \theta) \sin \theta d\theta \quad (10)$$

which by virtue of (8) and (2) yields

$$m_u = -\frac{1}{2} \int_0^\pi c_u^2 \sin \theta d\theta \quad (11)$$

Let us now recall the main conclusions emerging from the Bondi's approach.

1. If  $\gamma$ ,  $M$  and  $N$  are known for some  $u = a(\text{constant})$  and  $c_u$  (the news function) is known for all  $u$  in the interval  $a \leq u \leq b$ , then the system is fully determined in that interval. In other words, whatever happens at the source, leading to changes in the field, it can only do so by affecting  $c_u$  and viceversa. At the light of this comment the relationship between news function and the occurrence of radiation becomes clear.
2. As it follows from (11), the mass of a system is constant if and only if there are no news.

In the next section we calculate the rate of precession of a gyroscope at rest in the frame of (1)

### 3 The gyroscopic precession

Let us start by defining the vorticity vector, which as usual is given by (in relativistic units)

$$\begin{aligned}\omega^\alpha &= \frac{1}{2\sqrt{-g}}\epsilon^{\alpha\beta\gamma\delta}u_\beta\omega_{\gamma\delta} \\ &= \frac{1}{2\sqrt{-g}}\epsilon^{\alpha\beta\gamma\delta}u_\beta u_{\gamma,\delta}\end{aligned}\tag{12}$$

where the vorticity tensor is given by

$$\omega_{\alpha\beta} = u_{[\alpha;\beta]} - \dot{u}_{[\alpha}u_{\beta]}\tag{13}$$

and  $u_\beta$  denotes the four-velocity vector.

Now, for an observer at rest in the frame of (1), the four-velocity vector has components

$$u_\alpha = \left(A, \frac{e^{2\beta}}{A}, \frac{Ur^2e^{2\gamma}}{A}, 0\right)\tag{14}$$

with

$$A \equiv \left(\frac{V}{r}e^{2\beta} - U^2r^2e^{2\gamma}\right)^{1/2}\tag{15}$$

using (14) and

$$\sqrt{-g} = e^{2\beta}r^2 \sin \theta\tag{16}$$

in (12), we easily obtain

$$\omega^\alpha = (0, 0, 0, \omega^3) \quad (17)$$

with

$$\begin{aligned} \omega^3 = & -\frac{e^{-2\beta}}{2r^2 \sin \theta} \left\{ 2\beta_\theta e^{2\beta} - \frac{2e^{2\beta} A_\theta}{A} - (Ur^2 e^{2\gamma})_r \right. \\ & \left. + \frac{2Ur^2 e^{2\gamma}}{A} A_r + \frac{e^{2\beta} (Ur^2 e^{2\gamma})_u}{A^2} - \frac{Ur^2 e^{2\gamma}}{A^2} 2\beta_u e^{2\beta} \right\} \end{aligned} \quad (18)$$

and for the absolute value of  $\omega^\alpha$  we get

$$\begin{aligned} \Omega \equiv (-\omega_\alpha \omega^\alpha)^{1/2} = & \frac{e^{-2\beta-\gamma}}{2r} \left\{ 2\beta_\theta e^{2\beta} - 2e^{2\beta} \frac{A_\theta}{A} - (Ur^2 e^{2\gamma})_r \right. \\ & \left. + 2Ur^2 e^{2\gamma} \frac{A_r}{A} + \frac{e^{2\beta}}{A^2} (Ur^2 e^{2\gamma})_u - 2\beta_u \frac{e^{2\beta}}{A^2} Ur^2 e^{2\gamma} \right\} \end{aligned} \quad (19)$$

Feeding back (3)–(6) into (19) and keeping only the leading term, we obtain

$$\Omega = -\frac{1}{2r} (c_{u\theta} + 2c_u \cot \theta) + O(r^{-n}) ; \quad n > 1 \quad (20)$$

Now, since  $\Omega$  measures the rate of rotation with respect to proper time of world lines of points at rest in the frame of (1), relative to the local compass of inertia, then  $-\Omega$  describes the rotation of the compass of inertia (“the gyroscope”) with respect to reference particles at rest in the frame of (1) (see [11] for detailed discussion on this point).

Therefore, up to order  $1/r$ , the gyroscope will precess as long as the system radiates ( $c_u \neq 0$ ). Observe that if

$$c_{u\theta} + 2c_u \cot \theta = 0 \quad (21)$$

then

$$c_u = \frac{F(u)}{\sin^2 \theta} \quad (22)$$

which implies

$$F(u) = 0 \implies c_u = 0 \quad (23)$$

in order to insure regularity conditions, mentioned above, in the neighbourhood of the polar axis ( $\sin \theta = 0$ ). Thus the leading term in (20) will vanish if and only if  $c_u = 0$ .

If the system radiates during an interval of time  $\Delta u$ , then the change of orientation of the gyroscope, for that period, is given by

$$\Delta\phi = -\Omega\Delta u \left( \frac{V}{r} e^{2\beta} - U^2 r^2 e^{2\gamma} \right)^{1/2} \quad (24)$$

or, up to terms of order  $1/r$

$$\Delta\phi \approx \frac{1}{2r} (c_{u\theta} + 2c_u \cot \theta) \Delta u \quad (25)$$

Let us now consider the particular case of a quadrupole radiation in the linear approximation. If the quadrupole moment of the source is  $Q(u)$ , then it can be shown (see eqs. (86)–(91) in [7]) that in the linear approximation

$$c = \frac{1}{2} Q_{uu} \sin^2 \theta \quad (26)$$

and

$$-m_u = \frac{2}{15} Q_{uuu}^2 \quad (27)$$

Thus

$$\Omega = -\frac{1}{2r} \sin 2\theta Q_{uuu} + O(r^{-n}) \quad (28)$$

or

$$\Omega = \sqrt{\frac{15}{8}} \frac{\sin 2\theta}{r} (-m_u)^{1/2} + O(r^{-n}) \quad (29)$$

linking directly the rate of precession to the rate of loss of mass.

## 4 The gyroscopic precession of order $\frac{1}{r^2}$

Let us now consider the next order ( $\frac{1}{r^2}$ ). We easily obtain:

$$\begin{aligned} \Omega = & -\frac{1}{2r} (c_{u\theta} + 2c_u \cot \theta) \\ & + \frac{1}{r^2} [M_\theta - M(c_{u\theta} + 2c_u \cot \theta) - cc_{u\theta} + 6cc_u \cot \theta + 2c_u c_\theta] \end{aligned} \quad (30)$$

Observe that the order  $\frac{1}{r^2}$  contains, beside the terms involving  $c_u$ , a term not involving news ( $M_\theta$ ). Let us now assume that initially (before some  $u = u_0 = \text{constant}$ ) the system is static, in which case

$$c_u = 0 \quad (31)$$

which implies, because of (9)

$$M_\theta = 0 \quad (32)$$

and  $\Omega = 0$  (actually, in this case  $\Omega = 0$  at any order) as expected for a static field (for the electrovacuum case however, this may change [12]). Then let us suppose that at  $u = u_0$  the system starts to radiate ( $c_u \neq 0$ ) until  $u = u_f$ , when the news vanish again. For  $u > u_f$  the system is not radiating although (in general)  $M_\theta \neq 0$  implying (see for example (9)) time dependence of metric functions (non-radiative motions [7]).

In the interval  $u \in (u_0, u_f)$  the leading term of the rate of precession of the gyroscope is given by (20).

For  $u > u_f$  there is a precession term of order  $\frac{1}{r^2}$  describing the effect of the tail of the wave on the gyroscope. This in turn provides “observational” evidence for the violation of the Huygens’s principle, a problem largely discussed in the literature (see for example [7],[9], [13] and references therein).

Putting aside the actual technical difficulties in performing such an experiment, it should be clear that the monitoring of the gyroscope in the interval ( $u < u_0$ ,  $u > u_f$ ) should, in principle, bring out, in a clear-cut way, the presence of gravitational radiation.

Finally, let us consider the particular case of a quadrupole radiation in the linear approximation. We obtain for this case (note a misprint in Eq.(87) in [7]).

$$\Omega = \sqrt{\frac{15}{8}} \frac{\sin 2\theta}{r} (-m_u)^{1/2} + \frac{1}{r^2} (-3Q_{uu} \sin 2\theta) \quad (33)$$

Therefore, for  $u > u_f$ , the rate of precession is controlled by the second time derivative of the quadrupole moment ( $Q_{uu}$ ).

## 5 Scenarios of radiation and estimations

Let us now present some rough estimates for  $\Omega$  or  $\Delta\phi$ , in different scenarios. Before doing that, some remarks are in order.



Since our intention here is just to provide orders of magnitude of the expected effect, we shall restrain ourselves to the quadrupole radiation case. Also, although Bondi approach implies axial symmetry, it is not clear that such symmetry is present in all examples below. This may be particularly true for the case of collisions and bremsstrahlung. In the other cases, specially in those of gravitational collapse and supernovae, axial symmetry is not a too stringent condition. In the same line of arguments, it should be mentioned that in some examples, particularly in binary systems, the signal may last for a too long duration. Since the convergence of the series requires (see [10] for details)

$$u < 2r \quad (34)$$

then, in those examples, the source should be very far from the gyroscope, in order to assure the convergence of the series expansion. At any rate we should insist on the point that the purpose of this section is not to propose specific scenarios for eventual experiments, but just to provide , however vague, orders of magnitude of the mentioned effect.

Now, in the case of quadrupole radiation (in the linear approximation) the rate of precession can be related to the rate of loss of mass of the source through (29). Then for the change of orientation of the gyroscope  $\Delta\phi$ , during an interval of time  $\Delta u$  we obtain using (24),

$$\Delta\phi = \sqrt{\frac{15}{8}} \frac{\sin 2\theta}{r} (-m_u)^{1/2} \Delta u \quad (35)$$

This last equation will be used to obtain estimations of  $\Delta\phi$ , whenever the rate of loss of mass (or the total radiated mass) and the time interval of radiation are available. In some examples, when there is not a characteristic time scale of the emission we give an estimate of  $\Omega$ , from (29). For simplicity the numerical factor and the trigonometric term, in (35) and (29), are put equal to one.

In the last three decades, a great deal of work has been done in identifying possible sources of gravitational radiation (see [14] , [15] and references therein). Here we shall present a selection of some of them, keeping in mind all reserves mentioned above:

1. Binary systems.

- For the binary system of two neutron stars proposed by Clark [15], at 10 Kpc, we obtain  $\Delta\phi \approx 2.2 \times 10^{-15}$ . This quantity is obtained for an emission time of 1 second, of a binary system emitting mass at a rate  $m_u \sim 10^{47} \text{ Joules/s}$ .
- For the pulsar 1937+214 (see [16]), one obtains  $\Omega \approx 1.23 \times 10^{-23} \text{ rad.s}^{-1}$ . The estimated value used for its angular velocity  $4033.8 \text{ rad/s}$  leads to a power of emission around  $2 \times 10^{29} \text{ watts}$ , taking into account the Landau's formula and considering a distance of  $2.5 \text{ Kpc}$  for the source. Identifying the power emitted by the system with the loss of mass  $m_u$ , we obtain the given above value of  $\Omega$ .
- Similar estimations may be done for the pulsar 1913+16 (see [17]), knowing that the power of this pulsar is  $6.4 \times 10^{23} \text{ watt}$ . The estimates yield, for a distance of  $5 \text{ Kpc}$ ,  $\Omega \approx 1.13 \times 10^{-26} \text{ rad.s}^{-1}$ .

## 2. Gravitational collapse and supernovae

- According to Shapiro [15], during the first bounce and rebound, the efficiency of gravitational radiation never exceeds  $10^{-3} M_\odot$ , then for an event at 10Kpc with a duration of the order of 1ms (free fall time), the maximum obtained  $\Delta\phi$  is of the order of  $3 \times 10^{-18}$ .
- The model of collapse proposed by Wilson [15], assumes a radiating energy (in the form of gravitational radiation) of the order of  $10^{-2} M_\odot$ , during an interval of time of  $100 M_\odot$  (in relativistic units). Then for one solar mass, at a distance of 10Kpc, one obtains  $\Delta\phi \approx 6.4 \times 10^{-18}$ .
- In the strongest massive star collapse proposed by Ostriker [15], the emission is of the order of  $10^{-1}$  solar masses during  $10^{-1} \text{ s}$ , at 1 Kpc. The resulting  $\Delta\phi$  is of the order of  $2.9 \times 10^{-15}$ .
- For a model of supernovae proposed by Braginsky and Rudenko [18], (an emission of  $10^{48} \text{ J/s}$ , during 1 ms at 15Mpc) one obtains  $\Delta\phi \approx 4.2 \times 10^{-21}$ .
- The characteristic parameters of a stellar collapse model proposed by Rees et al [19], are : an emission of  $10^{50} \text{ J/s}$  during  $5 \times 10^{-4} \text{ s}$ . In such a collapse there is a continuous frequency spectrum up

to  $\frac{1}{\tau_\rho}$ , being  $\tau_\rho \sim \sqrt{\pi G} \rho$  the characteristic time of the collapse for a final state density  $\rho$ . The variation of energy  $\Delta E$  goes like  $\Delta E \sim \frac{1}{30\pi} \frac{Q^2}{\tau_\rho^5}$  where  $Q$  is the quadrupole moment along the axis. For an event at 5 Kpc involving a star of  $6M_\odot$ , the resulting  $\Delta\phi$  is of the order of  $7 \times 10^{-17}$ .

- Finally, let us evaluate the emission of gravitational radiation in a supernovae event, leading to a neutron star, by estimating the total change of the gravitational quadrupole moment of the source. Recent estimations of the quadrupole moment of neutron stars [20], point to values of  $Q$  of the order of  $1.03 \times 10^{37} \text{kg.m}^2$ . If we assume that the presupernovae massive star has a quadrupole moment of the order of the sun ( $3.85 \times 10^{42} \text{kg.m}^2$ ) [21], then the total change of quadrupole moment is  $\Delta Q \approx 10^{42} \text{kg.m}^2$ . Taking into account (27),(35), then the obtained  $\Delta\phi$ , for an event at distance of 1 Mpc, during  $10^7 \text{years}$  (Kelvin-Helmholtz time scale for a star like the sun), is of the order of  $2.8 \times 10^{-28}$ .

### 3. Collisions and Bremsstrahlung.

- The emission, during collision of two neutron stars, given by Wilson [15] is of the order of  $10^{-3}M_\odot$  during an interval time of the order of  $50M_\odot$ . For an event at 10Kpc we obtain  $\Delta\phi \approx 1.14 \times 10^{-18}$ .
- In the case of two black holes collision, Detweiler [15] proposes an emission of  $10^{-3}M_\odot$ , with a collision time of the order of 10M. For an event at 10Kpc this leads to  $\Delta\phi \approx 6.4 \times 10^{-25}$ .
- For gravitational bremsstrahlung within the galaxy, Ostriker [15] suggest an emission of  $10^{-2}$  solar masses during one second, this yields a  $\Delta\phi$  of the order of  $3 \times 10^{-16}$ .

### 4. Other sources.

- Following a speculative discussion about the evolution of galactic nucleus, Ostriker [15] considers the possibility of destruction of  $10^8$  neutron stars at the center, in approximately  $10^{6.5}$  years, emitting  $10^{-2}$  solar masses in the form of gravitational radiation, an event

of this kind, at 1Mpc of distance would produce a total precession of the order of  $3 \times 10^{-11}$ .

## 6 Conclusions

We have seen so far that a gyroscope at rest in a Bondi frame will precess (up to order  $1/r$ ) as long as the system radiates, the rate of precession being given by (20).

Once the radiation stops (vanishing news) the gyroscope will continue to precess with a rate of rotation given by the second term of (30) with  $c_u = 0$ .

As can be seen from estimations done above, excluding the last example, and the example based on the evaluation of the change in the quadrupole moment, the most realistic scenarios point to  $\Delta\phi$  's ranging between  $10^{-15}$  and  $10^{-19}$ . We ignore how far are we, with the present technology, to the accuracy required for that kind of measurement. Nevertheless, we want to stress that it has been our main purpose here, just to bring out such effects in the context of Bondi formalism.

We would like to conclude with the following comment: observe that all along our discussion we have not made reference to specific bandwidths. This is so because all quantities in the Bondi approach, are not defined with respect to any specific frequency. In this sense eq.(11) (and all related quantities), have to be considered as integrated over all frequencies.

## Acknowledgment

It is a pleasure to thank Prof. Bondi for interesting comments.

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